Overall Summery:

In the modern theory of electromagnetism, a topological theory can be expressed in terms of an exterior differential system of two postulates:

1. Flux is Conserved:
2. Currents are conserved:

The thermodynamic field “intensities” is defined of inexact 1-form potentials A. The exact 3-form current density J is defined by 2-form density of thermodynamic field quantities G(D,H). The two form is associated with forces and the 2-form density G associated with sources. From these postulates you can roughly deduce two of the classical PDE’s:

1. = 0
2. = J

The “modern” and “classical” electromagnetic theory have the same base of PDE’s. The differences are due to the equations two thermodynamic categories that are topologically distinct. The 1-form of potentials can have non-unique but topologically closed components, . That do not contribute to the 2-form of intensities F, as . Similarly, the 2-form density of excitations can have n0n-unique but closed components , which don’t contribute to 3-form density of currents J as d. The non- uniqueness can appear as discontinuities in solution amplitude or as multi-values as envelope solutions.

A topological perspective of E&M demonstrates that the Maxwell’s electromagnetic theory is universal and goes beyond theory’s that impose geometric constraints. The topological perspective leads to quantum-like structures and topological defects such as charge quantum without the imposition of quantum mechanics.

In conventional electrodynamics when we use a potential 1-form A for F as the fundamental object the usual considerations of wanting the field theory to be invariant under local U(1) gauge changes dictate that A must define a U(1) connection 1-form. However, the group that plays the fundamental role in topological electromagnetism seems to be SL(4), not SO(2). Reduction from SL(M) to SO(3,1)(M) is an essential step from the standpoint of introducing gravitation into the model, or deducing gravitation from it. The use of a constitutive law can give a simple and direct route for effecting the reduction from SL(M) to SO(3,1)(M). However it lacks an immediate topological construction. This either suggests that one cannot find a completely topological formulation for electromagnetism and must eventually resort to geometrical axioms or that we need to look for a more topological basis for the constitutive law.

What is a Gauge:

In 1918 Weyl treated Einstein’s general theory of relativity as if the Lorentz symmetry was a n example of global symmetry only mapped to local coordinates ad definable. So the magnitude of a vector is not a absolute quantity but depends on its space-time position or gauge invariance.:

If a gauge change is introduced by a multiplicative scaling factor S(x) with equals unity at x:

If a vector is constant under location change, then

Upon moving the magnitude changes by the amount

Weyl identified with the electromagnetic potential . This was rejected by Einstein because if the Compton wavelength defined by the mass of a particle, were dependent on position it would violate special relativity. So the Gauge change died off. However Gauge invariance survived because with arbitrary potentials, Maxwell’s equations form the E,B,H and D fields have build in symmetry the such potentials can simplify calculations. But the arbitrariness of a potential does not exit in quantum mechanics.